



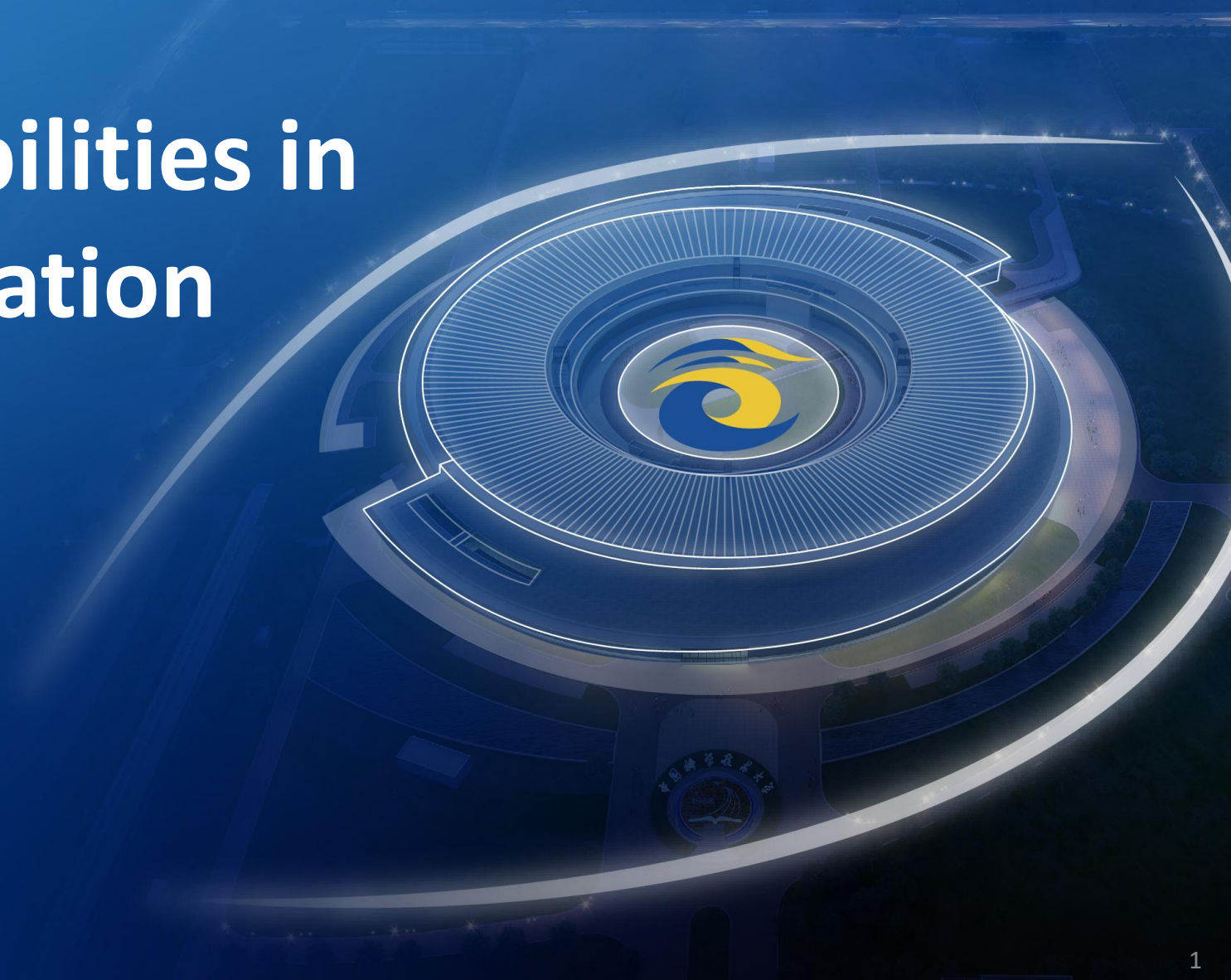
University of Science and Technology of China
National Synchrotron Radiation Laboratory

Asian Forum for Accelerators and Detectors
13-17 October 2026 Shanghai, China

Zero-Mode Instabilities in Synchrotron Radiation Storage Rings

Tianlong He (NSRL, USTC)

2026.10.14



Outline

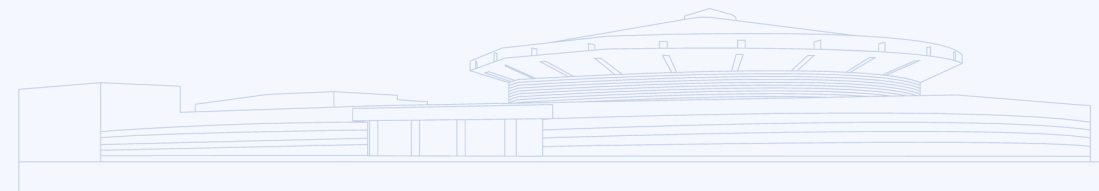
01 Introduction

- Storage rings & double RF systems
- Recap the well-known instabilities

02 Two types of zero-mode instabilities

- Driven by the SC harmonic cavity with high $Q_L > 10^8$
- Driven by the main cavity with PI feedback

03 Summary



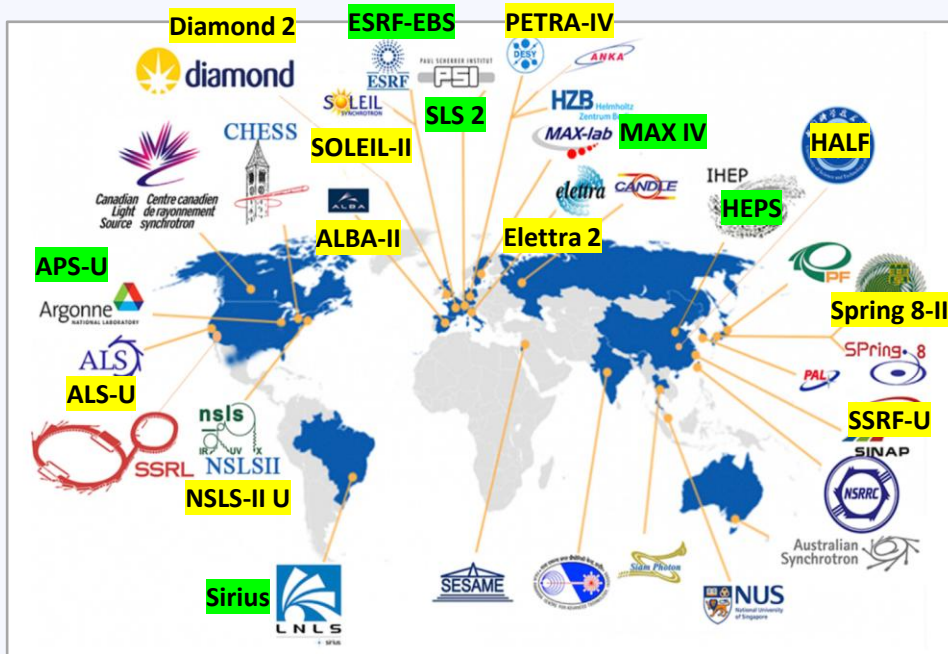
New generation synchro. rad. storage rings



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More than sixteen 4th generation light source facilities are in operation or under construction/planning worldwide.

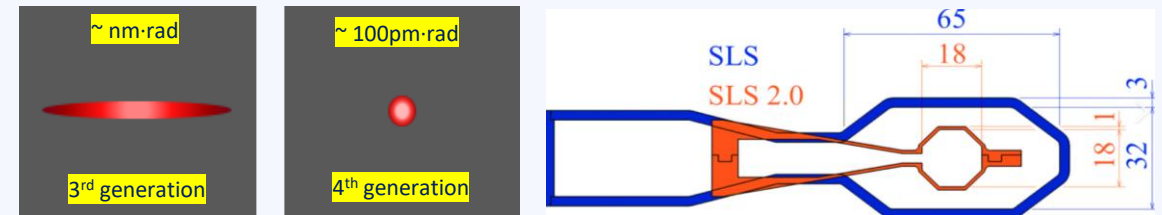
Ultralow emittance (stronger focusing, smaller dispersion); smaller vacuum chamber; high beam current.



In China:

HEPS

HALF



Intra-beam scattering , Touschek effect
Instabilities: single bunch, multi-bunch
Heating issues: parasitic energy loss



More challenging collective effects

Hefei Advanced Light Facility

Circumference 480 m

Beam energy 2.2 GeV

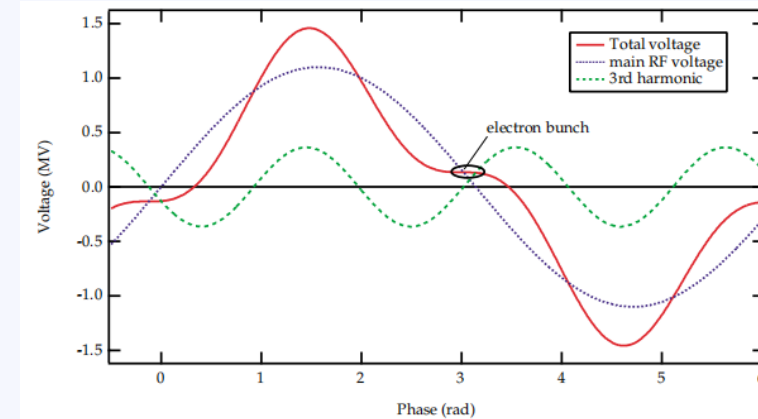
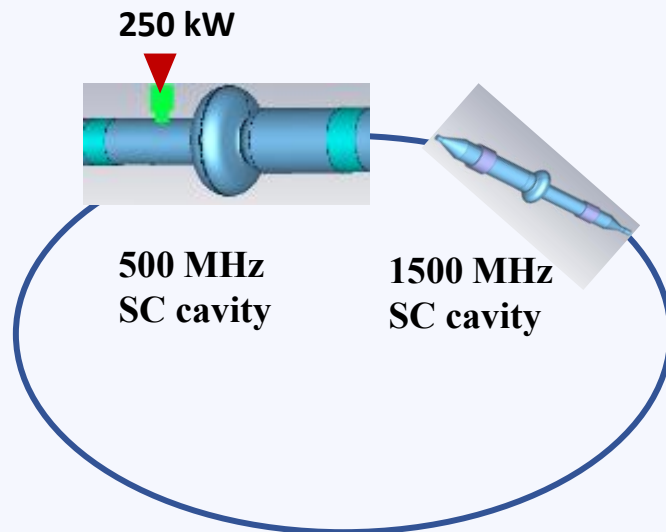
Beam current 350 mA

Natural emittance 86 pm rad

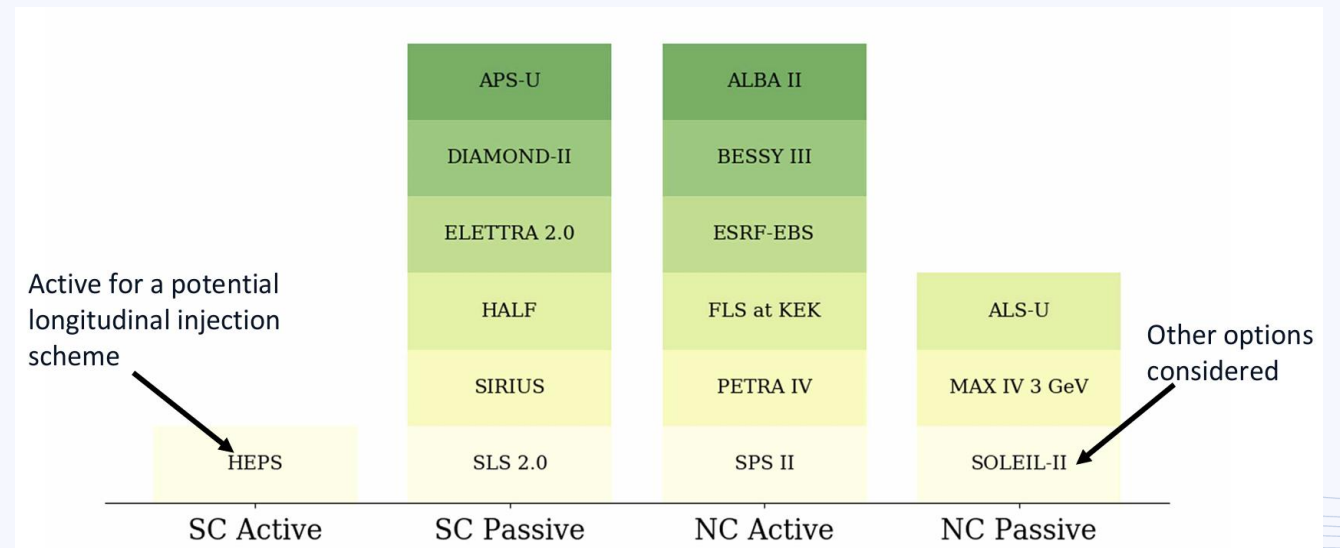


- Double RF system:
main cavity & **harmonic cavity**

HALF double RF system Configuration



J. M. Byrd, et al., PRAB, 2001

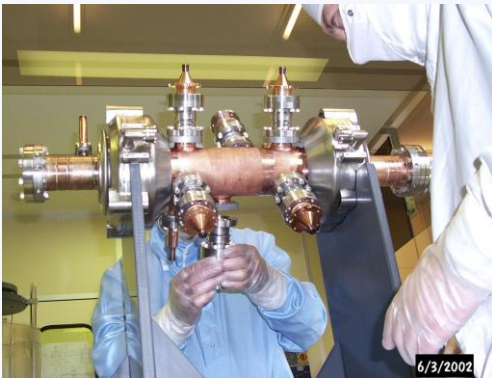


Courtesy Francis Cullinan (FLS2023)

Recap the superconducting harmonic cavities

In 3rd and 4th generation synchrotron storage rings:

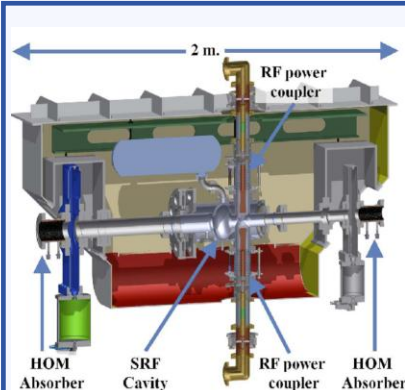
SLS/ELETTRA/SSRF; SLS 2/ELETTRA 2/Diamond 2/Sirius/APS-U/HEPS/HALF



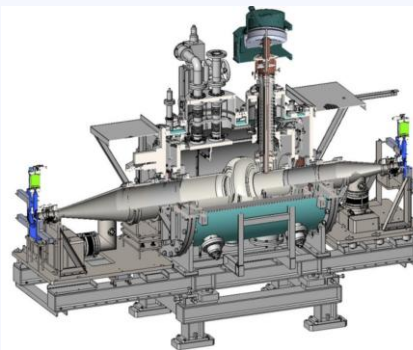
SLS/ELETTRA, SLS 2/ELETTRA 2
Passive, ~ 1500 MHz @ 4K
Two cell, w/o coupler
Cu wall with Nb coating



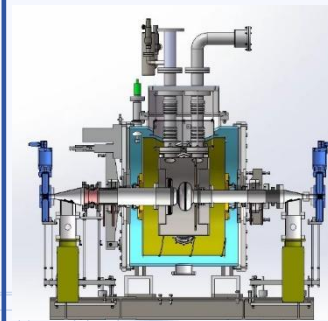
SSRF/Diamond 2/ Sirius
Passive,
~ 1500 MHz @4K
Two cell, w/o coupler



APS-U, passive,
~1408 MHz@2K
Single cell,
w/ two couplers



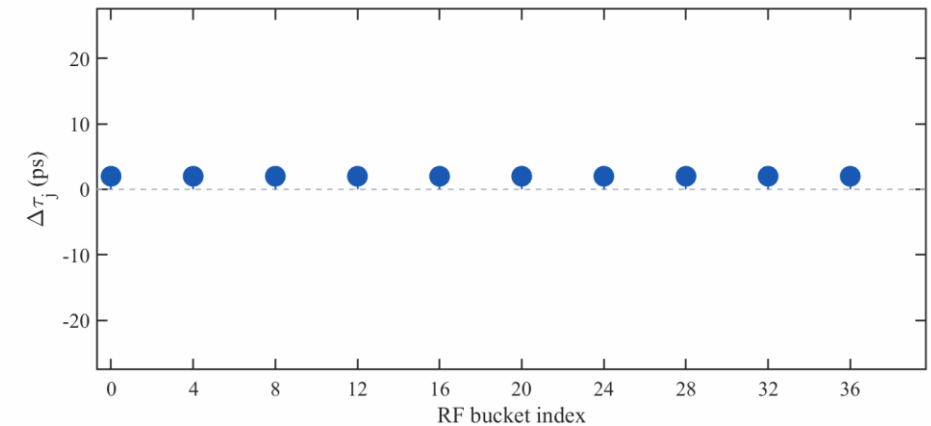
HEPS, active, ~500
MHz @ 4K
Single cell,
w/ one coupler



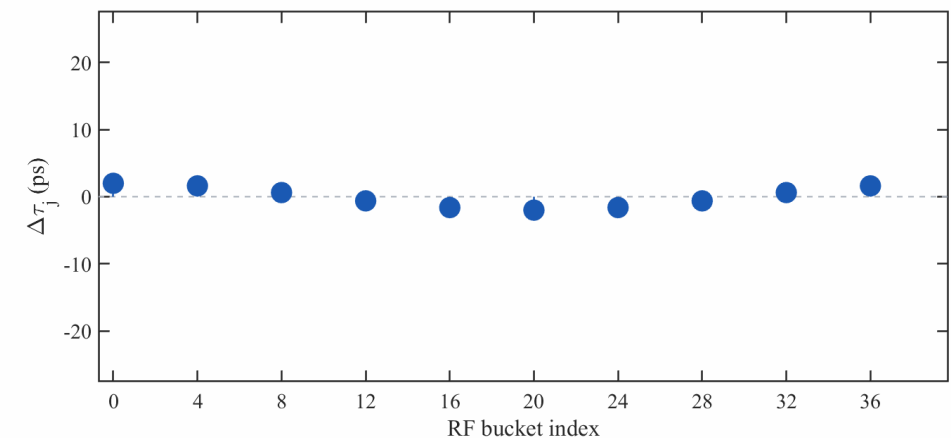
HALF, passive,
~1500 MHz@4K
Single cell,
w/o coupler

- Cavity fundamental mode impedance
- Of primary interest are the low-order coupled-bunch modes: 0, ± 1
- Mode 0: all bunches oscillate in phase, with zero phase difference
- Mode ± 1 : adjacent bunches have a phase difference of $2\pi/N$, N is the number of bunches

Mode 0 instability oscillation



Mode ± 1 instability oscillation



Recap the well-known instabilities

- **AC Robinson**
- DC Robinson
- D-Q coupled Robinson
- Mode 1 & PTBL
- D-mode (cavity mode)
- Mode 0 (beam-cavity-LLRF)

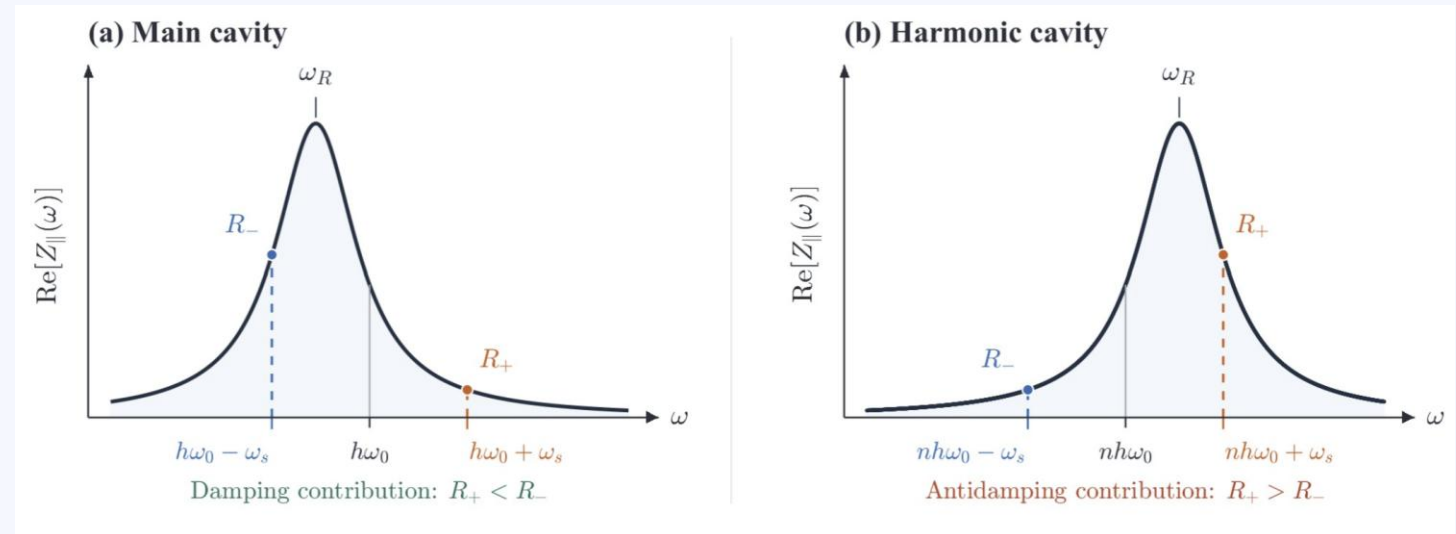
K. W. Robinson, CEAL-1010 (1964).

A. W. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators (Wiley, New York, 1993), p. 162.

A. Gamelin et al., PRAB 28, 054401 (2025).

In Chao's book

$$\text{Growth rate: } \tau^{-1} \propto [\text{Re}Z_{\parallel}(\omega_{1,3} + \omega_s) - \text{Re}Z_{\parallel}(\omega_{1,3} - \omega_s)]$$



The MC is negatively detuned, which damps the AC Robinson instability.

The HC is positively detuned, which drives the AC Robinson instability.

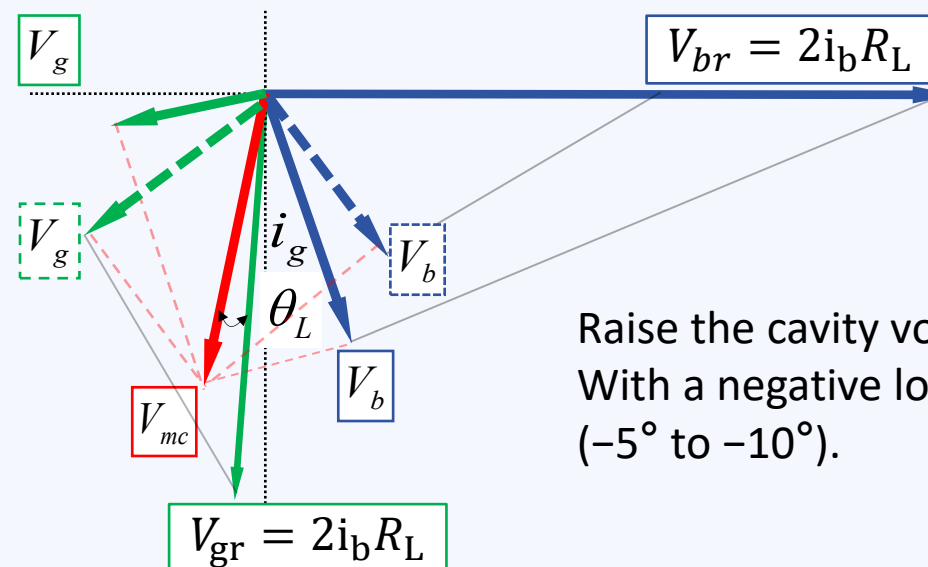
Recap the well-known instabilities

- AC Robinson
- **DC Robinson**
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T. Yamaguchi et al., PRAB 26, 044401 (2023).
F. Pérez et al., Proc. IPAC'24, WEPR76 (2024).
S. Sakanaka et al., Proc. IPAC'25, WEPS007 (2025)

Phasor diagram for the main cavity voltage

As the beam current increases, the detuning increases, and V_g gradually shifts from the third quadrant toward the second quadrant.



Raise the cavity voltage
With a negative loaded angle
(-5° to -10°).

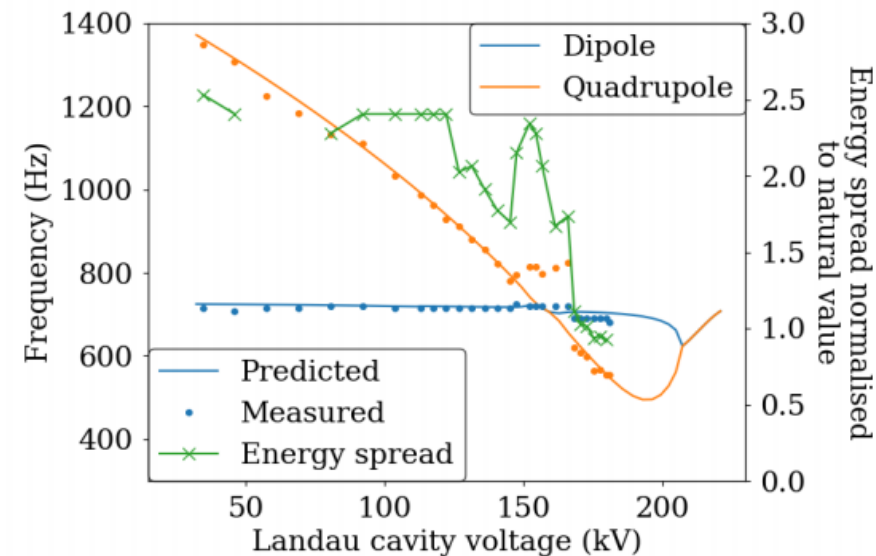
- Only the generator voltage V_g is capable of focusing the beam
- Focusing: V_g lies in the third quadrant
- Defocusing: V_g lies in the second quadrant

Recap the well-known instabilities

- AC Robinson
- DC Robinson
- **D-Q coupled Robinson**
- Mode 1 & PTBL
- D-mode (cavity mode)
- Mode 0 (beam-cavity-LLRF)

R. A. Bosch et al., PRST-AB 4, 074401 (2001).
F. J. Cullinan et al., Proc. IPAC'20, WEVIR05 (2020).
M. B. Alves, PRAB 28, 034401 (2025).

MAX-IV 3 GeV ring

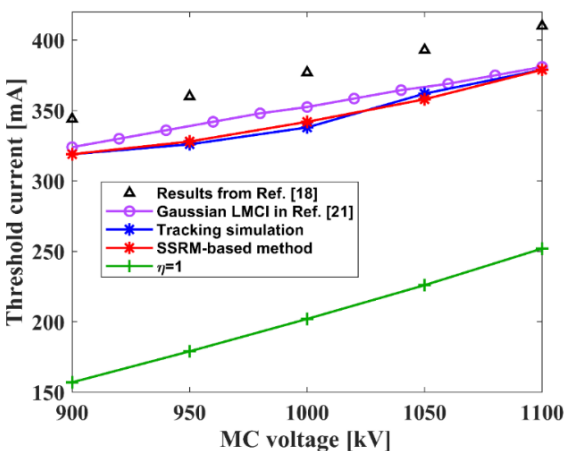


F. J. Cullinan et al., IPAC2020

- Dipole mode: bunch-centroid oscillation;
- Quadrupole mode: bunch-length oscillation
- Triggered when the two mode frequencies coincide
- Mode 0 damper is a must

Recap the well-known instabilities

- AC Robinson
- DC Robinson
- D-Q coupled Robinson
- Mode 1 & PTBL
- D-mode (cavity mode)
- Mode 0 (beam-cavity-LLRF)



J.C. Xiao, et al., NIMA 2026

Year	Progress	References
2018	Vlasov-based theory, Mode 1, imaginary part of Imp.	M. Venturini, PRAB 21
2022	Tracking simulation , description and definition of PTBL	T. He, et al., PRAB 25 (a)
2022	Centroid perturbation theory, approx. threshold formula	T. He, et al., PRAB 25 (b)
2024	Experimental observation	F. Cullinan, et al., PRAB 27
2025	ALBuMS - an open-source Python package, fast tool	A. Gamelin, et al., PRAB 28
2025	Lebedev equation, Gaussian LMCI, pycolleff	M. B. Alves, et al., PRAB 28
2025	PTBL in triple rf systems, SSRM	J.C. Xiao, et al., PRAB 28
2026	SSRM-based fast calculation of PTBL	J.C. Xiao, et al., NIMA

A common instability challenge in limiting bunch lengthening in 4th generation storage rings

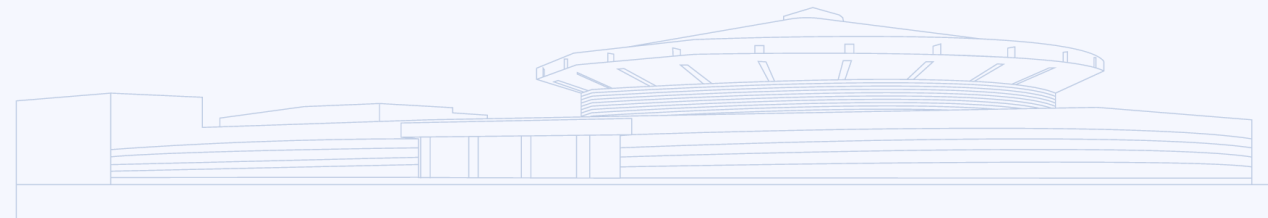
- **PTBL/Mode 1:** imaginary HC impedance driven; low frequency (no Landau damping); scales with $R/Q \cdot I_0$
- **Mitigation:** choose a low- R/Q cavity scheme → room-temperature TM020 (low R/Q) / superconducting TM010 (less cavities)
- **Calculation methods:** tracking simulation & linearized Vlasov equation
- **Accurate calculation:** centroid + bunch-length perturbations together

Recap the well-known instabilities



- AC Robinson
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- D-Q coupled Robinson
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- D-mode (cavity mode)
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Next, we will focus on...





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D-mode instability

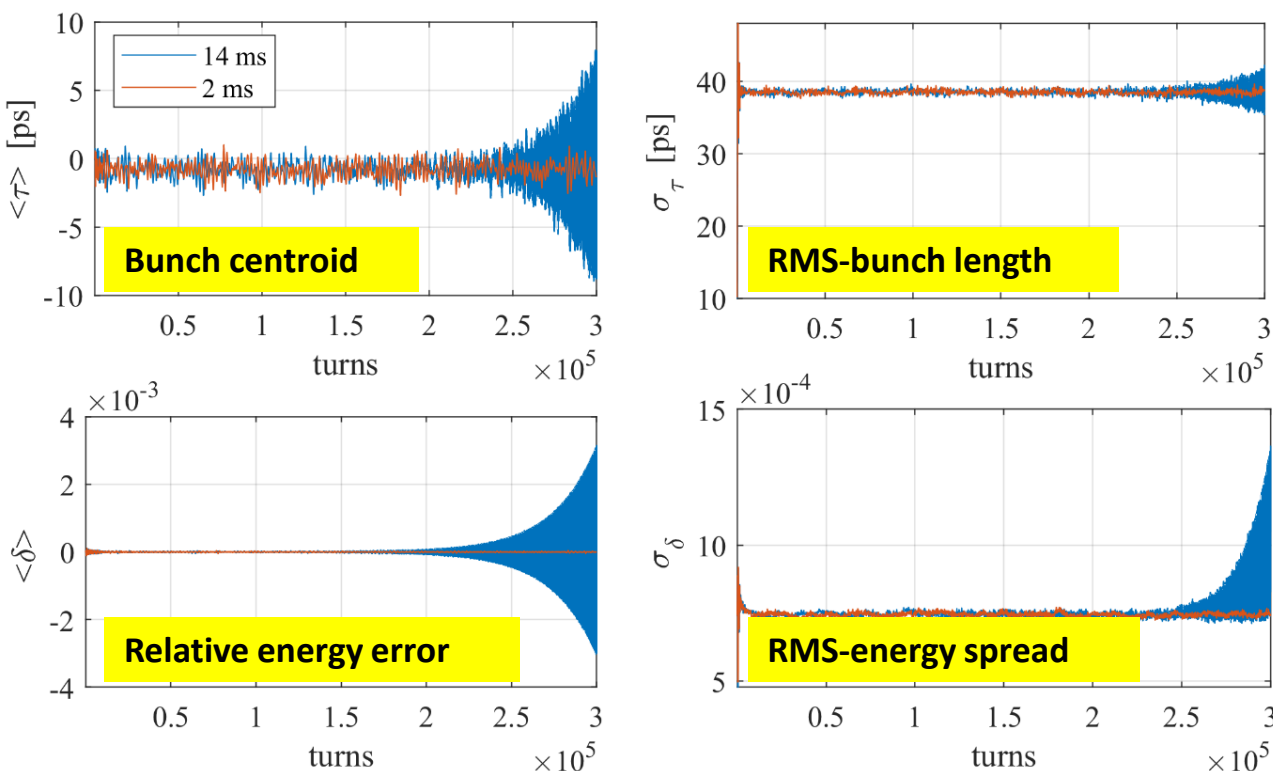
Driven by the high- Q_L fundamental mode of
a superconducting harmonic cavity

- The shorter the radiation damping time is set, the faster the instability grows.
- This is related to the ultra-high Q_L value; the oscillation frequency is slightly lower than the harmonic cavity detuning frequency.

HALF 2.2 GeV ring @40 mA, HHC $\Delta f_r=5.8$ kHz

TABLE I. Main parameters of the HALF storage ring used for the following calculations.

Parameter	Symbol	Value
Beam energy	E	2.2 GeV
Ring circumference	C	479.86 m
Assumed beam current	I_0	40 mA
Longitudinal damping time	τ_z	14 ms
Momentum compaction	α_c	9.4×10^{-5}
Harmonic number	h	800
Energy loss per turn	U_0	400 keV
MC voltage	V_{rf}	1.2 MV
MC normalized shunt impedance	R_1/Q_1	45 Ω
MC loaded quality factor	Q_1	1.14×10^5
MC detuning	Δf_{r1}	-700 Hz
PSHC harmonic order	n	3
PSHC normalized shunt impedance	R_n/Q_n	39 Ω
PSHC loaded quality factor	Q_n	2×10^8

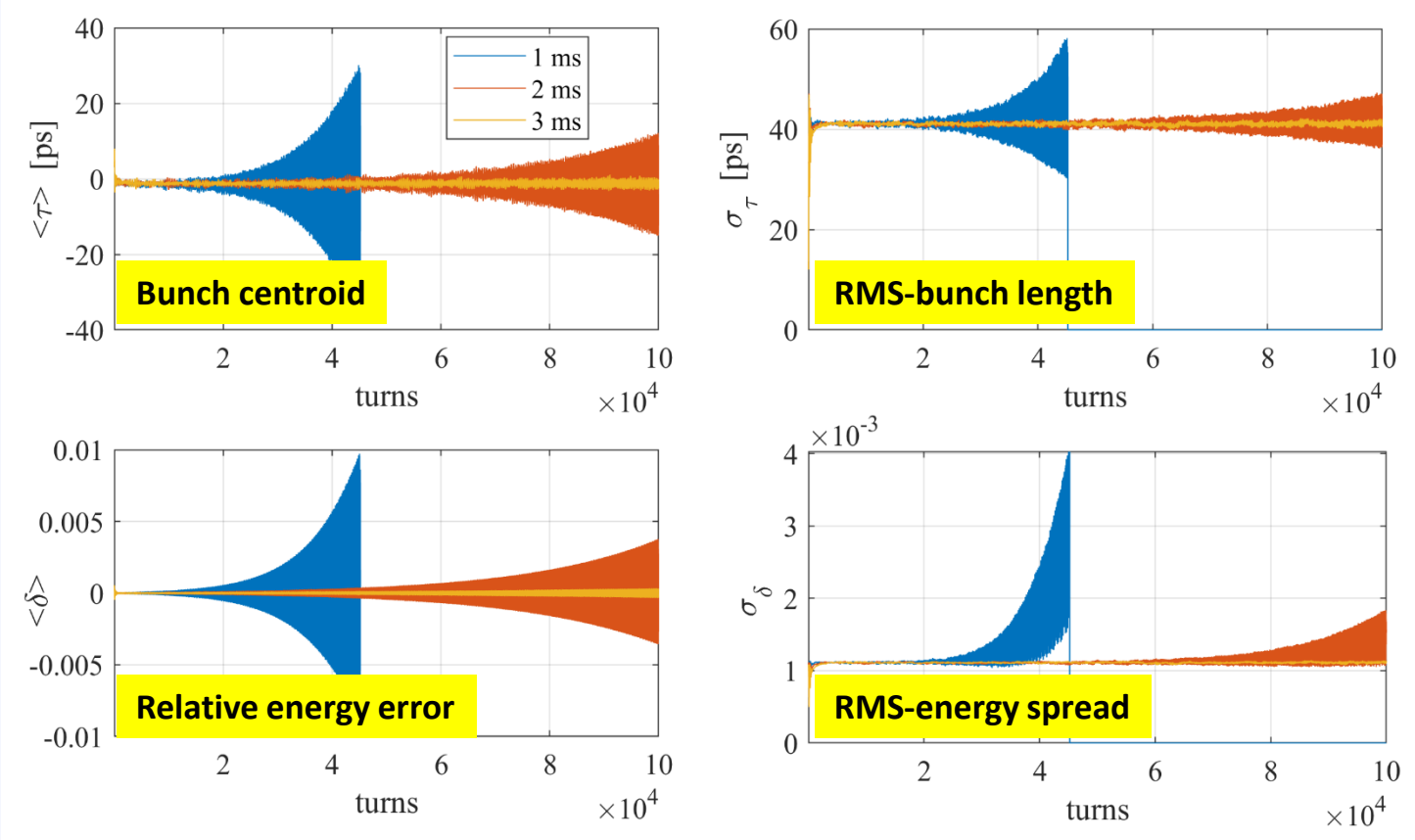


Self-consistent multi-macroparticle, multi-bunch tracking simulation

- The shorter the radiation damping time is set, the faster the instability grows.
- This is related to the ultra-high Q_L value; the oscillation frequency is slightly lower than the harmonic cavity detuning frequency.

SSRF 3.5 GeV ring @200 mA, HHC $\Delta f_r=19$ kHz

Parameters	Value
Energy	3.5 GeV
Current	200 mA
Circumference	432 m
Radiation loss	1.7 MeV
Harmonic number	720
Momentum compaction	4.2E-4
Radiation damping	3 ms
Energy spread	1.11E-3
Main voltage	4.5 MV
MC R/Q	44.5 * 3 Ohm
MC QL	1.6E5
MC detuning frequency	-2.9 kHz
HHC R/Q	88 Ohm
HHC QL	2e8



Some clues for the D-mode Robinson

➤ Observed in 3rd rings: ELETTRA/SLS/SSRF

• ELETTRA (2003)

M. Pedrozzi, et al., PAC2003

down to 100 mA. To keep constant the effect of the S3HC a voltage feedback has been implemented which acts on the tuning system of each cell. The feedback is opened at about 160 mA because, for lower currents, the S3HC frequency approaches the +3fs sideband; an instability is then observed causing beam loss.

① Approach +3fs

• SLS (2022): Beam loss at 100 mA when detuning < 23.6 kHz.

② Bosch's theory

• SSRF (2023): Beam loss at 200 mA when detuning is close to the resonance.

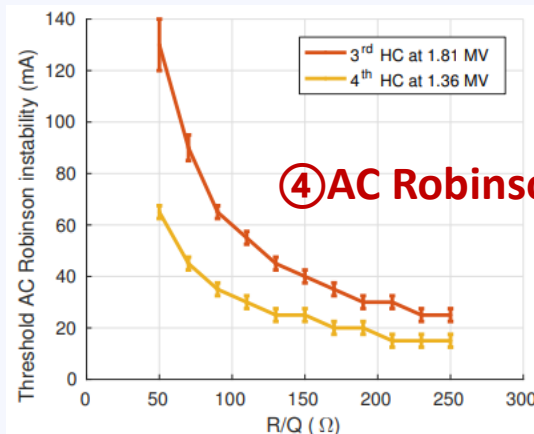
③ Approach 3frf

➤ Tracking for 4th rings: ESRF-EBS/SOLEIL II

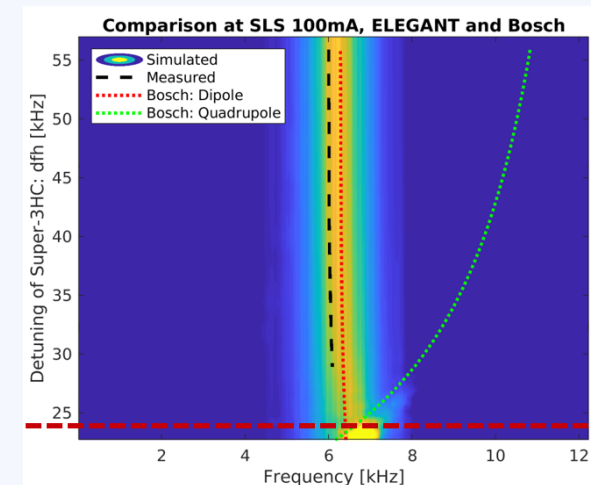
ESRF-EBS (2017)

N. Carmignani, et al., IPAC2017

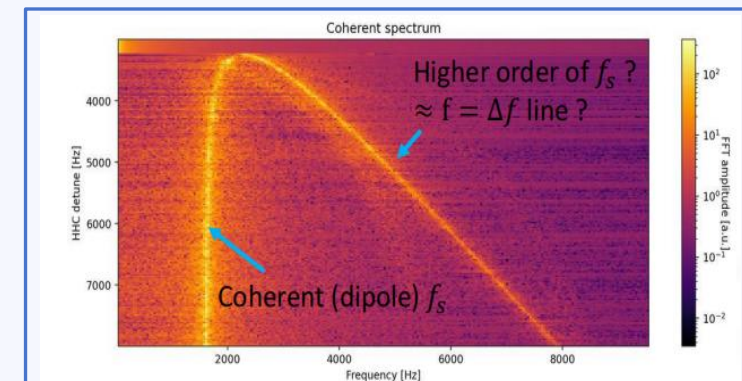
In case of low beam current, the passive harmonic cavity has to be tuned close to the resonance frequency to have the desired voltage and it results in being unstable. The longitudinal center of mass of the beam starts to oscillate at the detuning frequency of the cavity, which is of the same order of magnitude as the synchrotron frequency. At higher beam current, the passive harmonic cavity has to be detuned more and the system results in being stable.



④ AC Robinson



Lukas Stingelin @ HarmonLIP 2022



Alexis Gamelin @ HarmonLIP 2022

⑤ Bosch's theory or approach 1fs

Which one is correct? How to explain this instability?

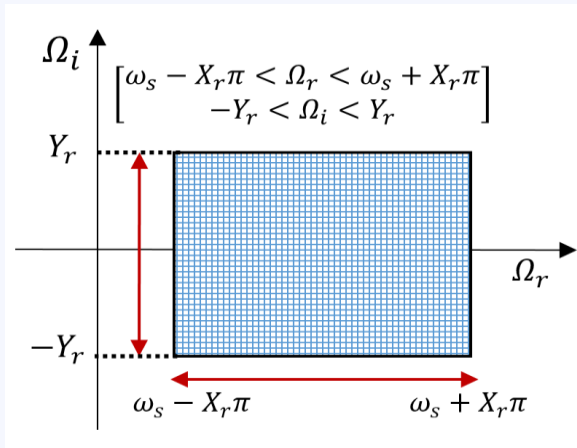
Constraints: ideal main cavity + passive harmonic cavity & uniform filling

- **Single-particle model**
- **Dispersion equation:**
$$\Omega^2 - \omega_s^2 + i \frac{\omega_0 I_0 \alpha_c}{2\pi E/e} \sum_{p=-\infty}^{\infty} \{pM\omega_0 Z(pM\omega_0) - (pM\omega_0 + \mu\omega_0 + \Omega)Z(pM\omega_0 + \mu\omega_0 + \Omega)\} = 0$$

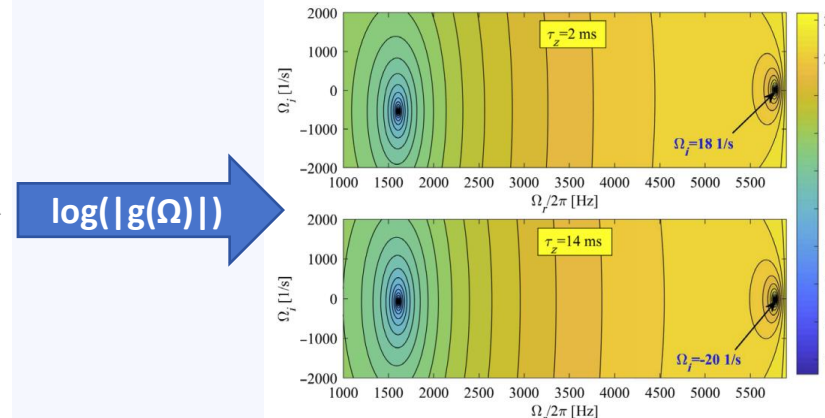
$$\Omega^2 - \omega_s^2 + i \frac{2\Omega}{\tau_z} + i \frac{\omega_0 I_0 \alpha_c}{2\pi E/e} \times \left\{ -i \frac{nh\omega_0 R f_r}{Q \Delta f_r} - \omega_p^+ Z_{\parallel}(\omega_p^+) - \omega_p^- Z_{\parallel}(\omega_p^-) \right\} = 0$$

ideal MC **rad. damping** HC impedance

- **Numerical root-finding algorithm**
→ minimize the absolute value
$$g(\Omega) = \Omega^2 - \omega_s^2 + i \frac{2\Omega}{\tau_z} + i \frac{\omega_0 I_0 \alpha_c}{2\pi E/e} \times \left\{ -i \frac{nh\omega_0 R f_r}{Q \Delta f_r} - \omega_p^+ Z_{\parallel}(\omega_p^+) - \omega_p^- Z_{\parallel}(\omega_p^-) \right\}$$



T. He, et al., PRAB 26 (2023)



HALF @ 40 mA, HHC@ Δfr=6 kHz

Find two minima:

- **One close to fs**: S-Mode, Beam-Mode
- **The other close to Δfr**: D-Mode, Cavity-Mode

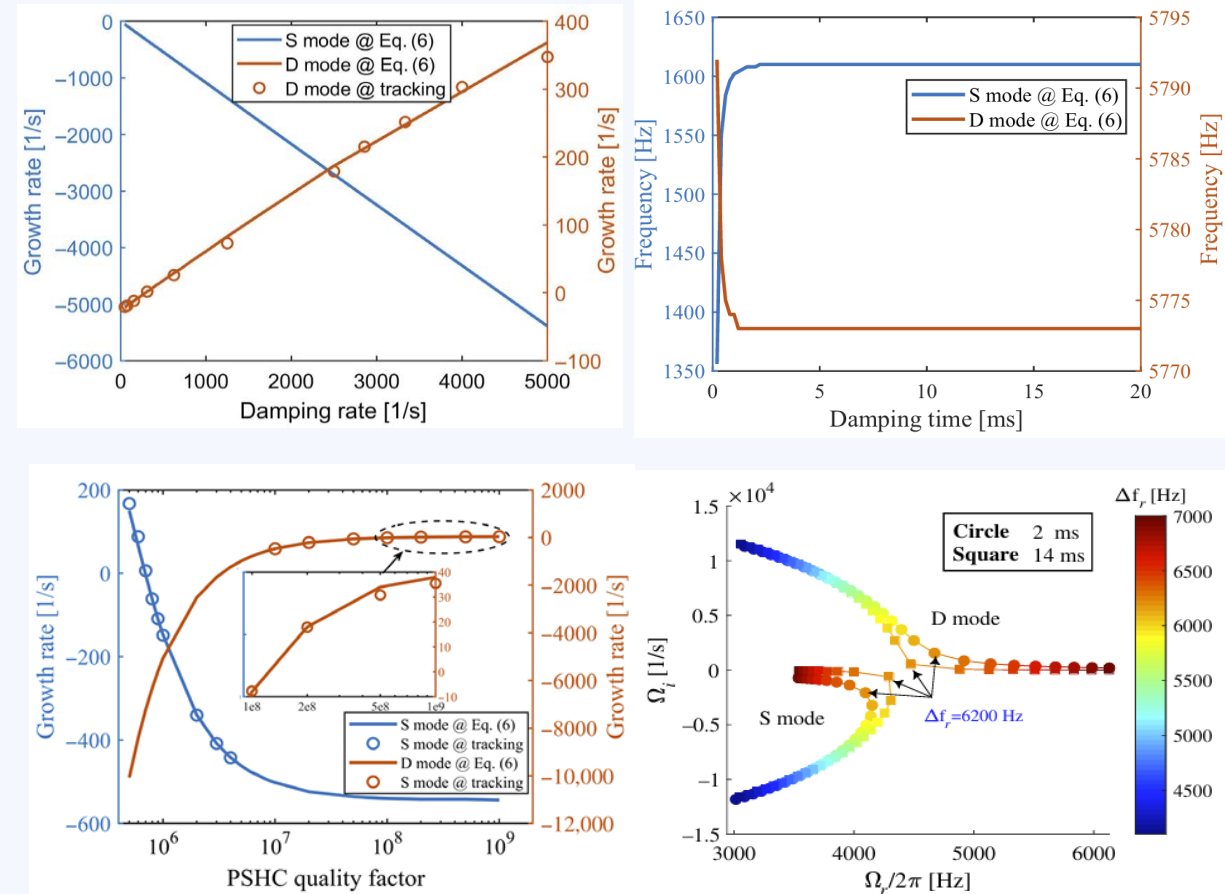
N. Towne and J.-M. Wang, "Spectrum of single bunch longitudinal dipole modes," *Physical Review E*, vol. 57, no. 3, p. 3461, 1998.

Impacting factors investigation: rad. damp. time; HHC Q_L & detuning

Insights:

- ① Shorter rad. damp. time $\rightarrow$ suppresses S-mode, promotes D-mode growth, and increases D-mode frequency.
- ② Larger $Q_L \rightarrow$ weakens S-mode, but enhances D-mode.
- ③ Smaller cavity detuning $\rightarrow$ promotes D-mode.
- ④ When D-mode approaches S-mode, they merge, causing S-D mode coupled instability with a large growth rate.
- ⑤ D-mode frequency is slightly below the cavity detuning frequency and well above the synchrotron frequency, so no Landau damping.

HALF 2.2 GeV ring @ 40 mA, HHC @ $\Delta f_r = 6$ kHz



Why does radiation damping promote the D-mode?

- The D-mode frequency characteristic allows us to simplify the dispersion equation and derive a simplified relation that includes the main cavity beam loading ($j = 1$).

$$\Omega^2 + i \frac{2\Omega}{\tau_z} - \omega_s^2 - i\xi \sum_{j=1,n} \{jZ_j(\omega_{pj}^+) - jZ_j(\omega_{pj}^-)\} = 0$$

- D-mode frequency: $\Omega_r = \overset{\text{HC detuning}}{\Delta\omega_{rn}} - \Delta\omega_1 \rightarrow \Delta\omega_1^2 - \left(\frac{\Delta\omega_{rn}}{2} + \frac{\zeta_1 - \omega_s^2}{2\Delta\omega_{rn}} - \frac{\xi n R_n \omega_{rn}}{8Q_n \Delta\omega_{rn}^2} \right) \Delta\omega_1 + \frac{\xi n \omega_{rn} R_n}{4\Delta\omega_{rn} Q_n} = 0$

- D-mode growth rate: $\Omega_i \approx \frac{b_n + b_1 - 2\Omega_r/\tau_z}{2\Omega_r - k_n - k_1}$

$$b_1 \approx \xi R_1 [\cos^2(\psi_-) - \cos^2(\psi_+)] < 0$$

$$b_n \approx \frac{\xi n R_n \omega_{rn}^2}{4Q_n^2 \Delta\omega_1^2} > 0 \quad k_n \approx \frac{\xi n R_n \omega_{rn}}{2Q_n \Delta\omega_1^2} > 0$$

- For SLS/ELETTRA: $\frac{k_n}{|k_1|} > \frac{n^2 R_n Q_1}{8Q_n R_1} \left(\frac{\omega_{r1}}{\Delta\omega_1 Q_1} \right)^2 > 10 \Rightarrow \Omega_i \approx \frac{b_n + b_1 - 2\Omega_r/\tau_z}{2\Omega_r - k_n}$

Normal, $2\Omega_r > k_n$
D-mode, $2\Omega_r < k_n$

Radiation damping does promote the D-mode!

- Theoretical Calculations and Tracking Simulations for ELETTRA/SLS/SSRF

[T. He, et al., PRAB 27, 064402 \(2024\)](#)

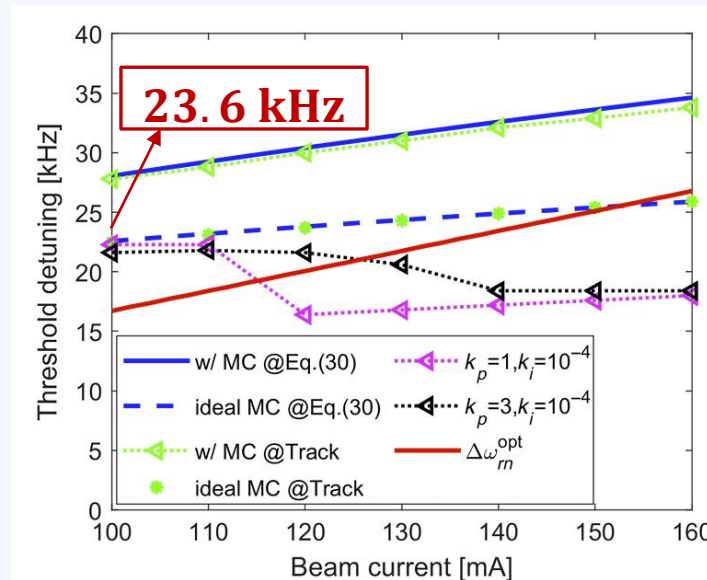
(ELETTRA's results not shown here, see details in this ref.)

- Without PI feedback: NC MC promotes the D-mode.
- With PI feedback: NC MC can suppress the D-mode to some extent.
- SC MC: weak effect on the D-mode.

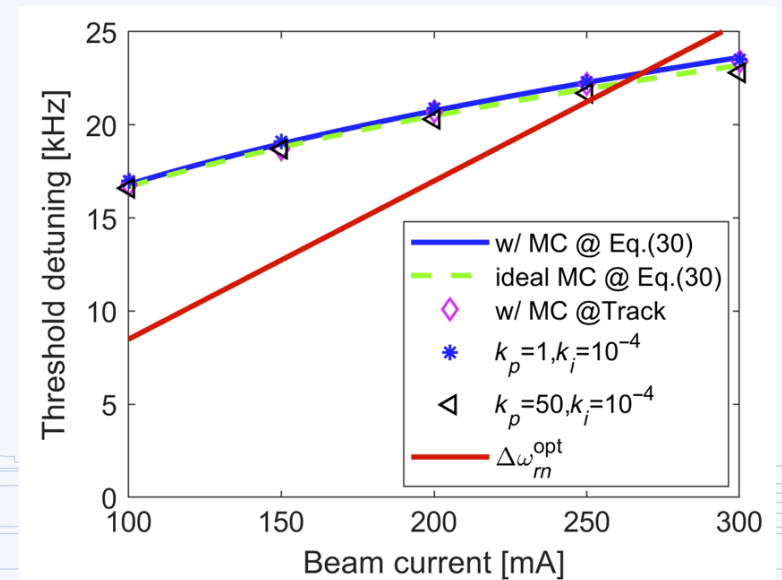
$$\Delta\omega_r^{\text{opt}} = \frac{F_{\text{opt}} I_0 \omega_r R}{V_h^{\text{opt}} Q}$$

Near-optimal detuning
frequency for bunch
lengthening

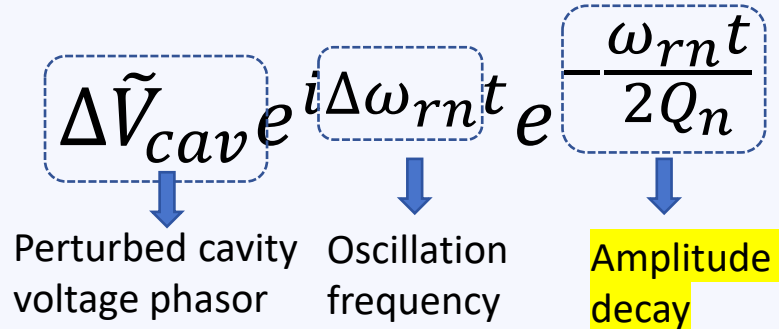
SLS MC: $Q_L \sim 10^4$



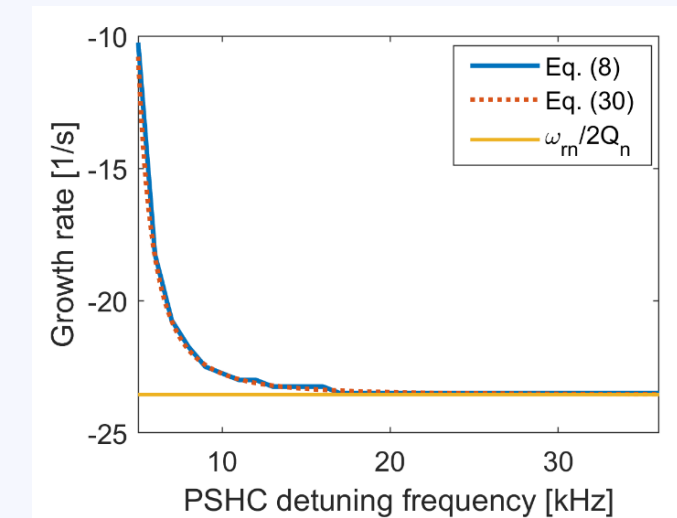
SSRF MC: $Q_L \sim 10^5$



Cavity voltage perturbation evolution as seen by the beam:



Normal-conducting: $QL \sim <1e4$; Superconducting: $QL \sim >1e8$



- Discussion: How to suppress this instability?

Cavity parameter optimization:

1. Moderately reduce the loaded Q_L
2. Choose R/Q properly according to the target beam current, avoiding operation at a low detuning frequency
3. Lower the MC voltage to reduce the HHC voltage requirement, thereby increasing the HHC detuning

Guidance for SC HC development:

Nb₃Sn & 4 K → 2 K: $Q_0, \uparrow Q_L \rightarrow \uparrow$
unfavorable for D-mode stabilization
→ Recommend Q_L reduced to $\sim 10^7$



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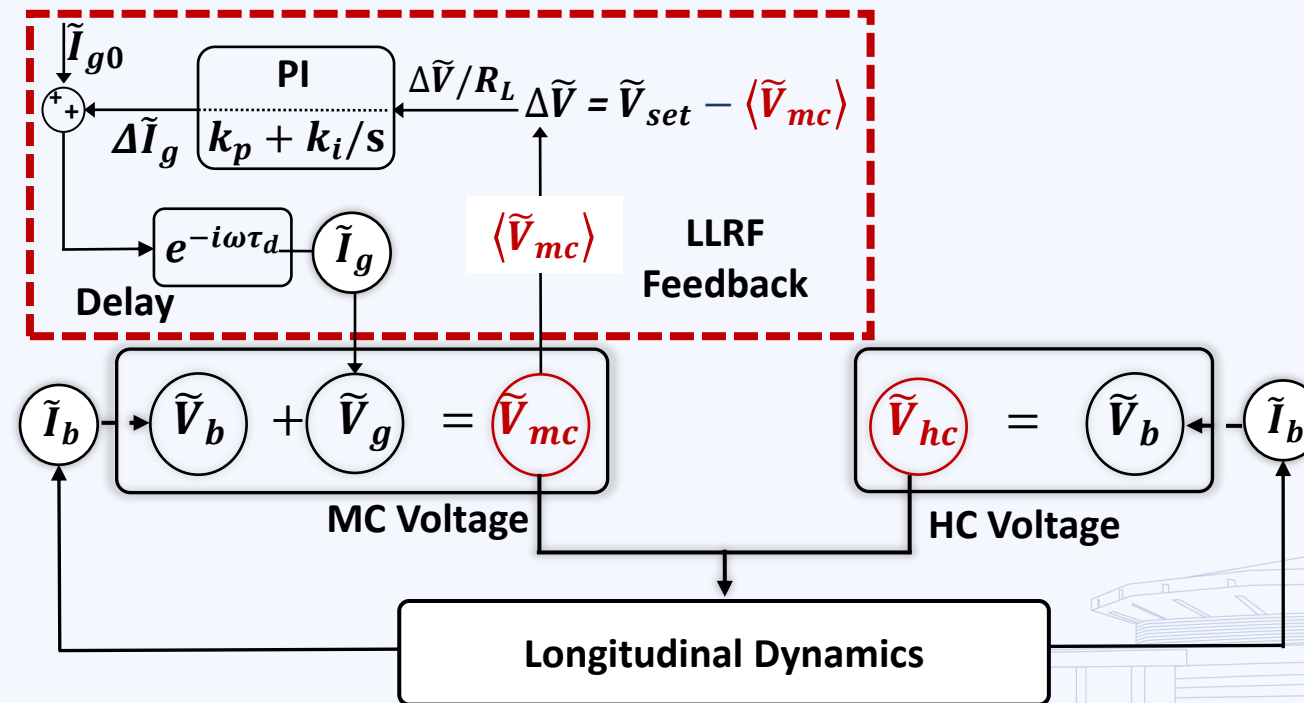
Zero-mode instability

Under the coupled scenario of the beam,
harmonic cavity, main cavity, and LLRF feedback

What problem are we dealing with?

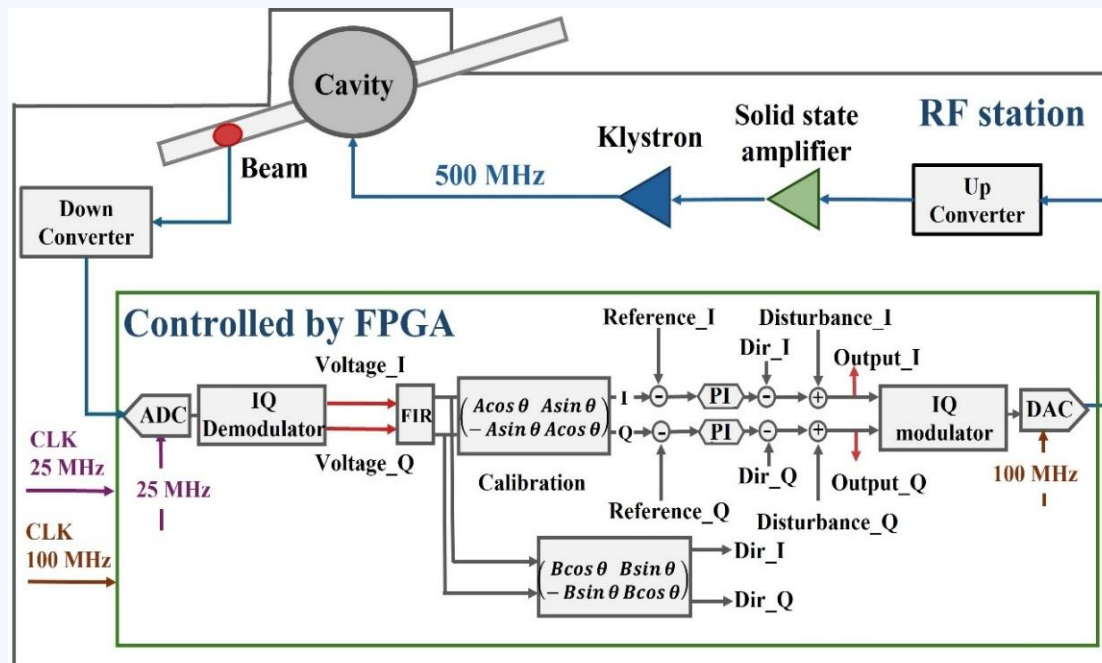
- **A complex multi-factor coupling problem:** beam–harmonic cavity–main cavity and its low-level RF feedback (PI controller)

Key issue: How to establish a more realistic mathematical model of the low-level RF feedback?



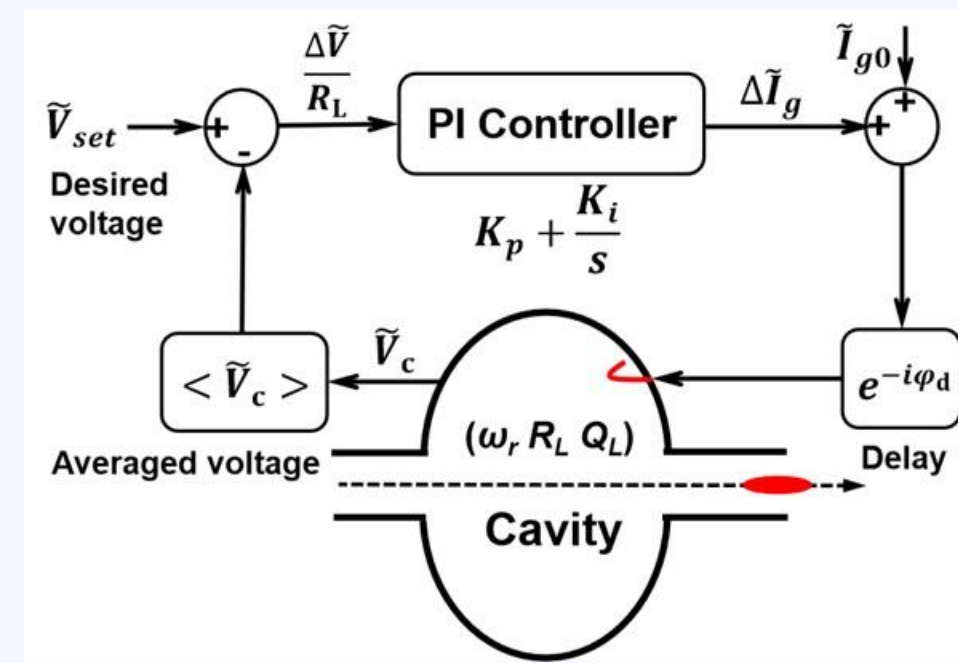
Modelling the PI controller

Pick up-Down conversion-ADC-I/Q Demodulation-**FIR Filter**---**PI**---IQ Modulation -DAC- Upconversion



SSRF- LLRF block diagram modified from the Fig.1 in
[Y. Xia, et al., NST 2019]

T. He et al., arXiv:2601.05541 (2026), Fig. 2.



PI gains: K_p, K_i ; Loop delay: τ_d

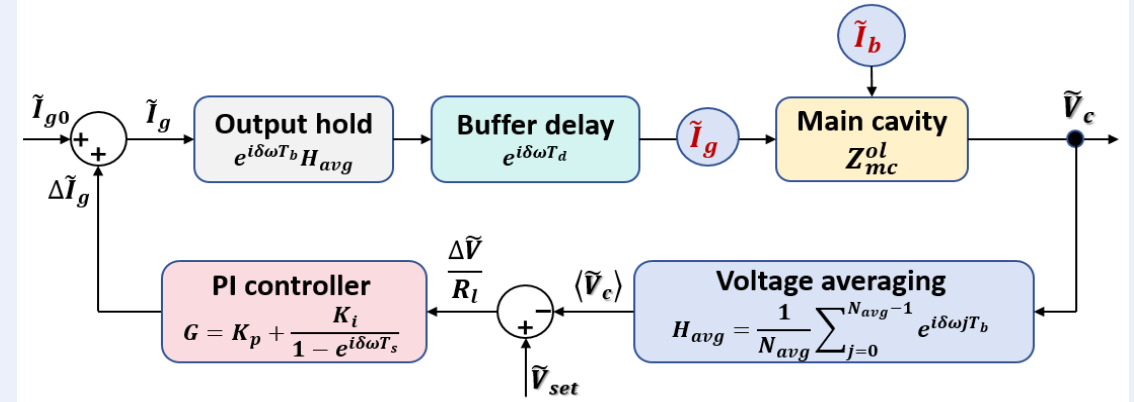
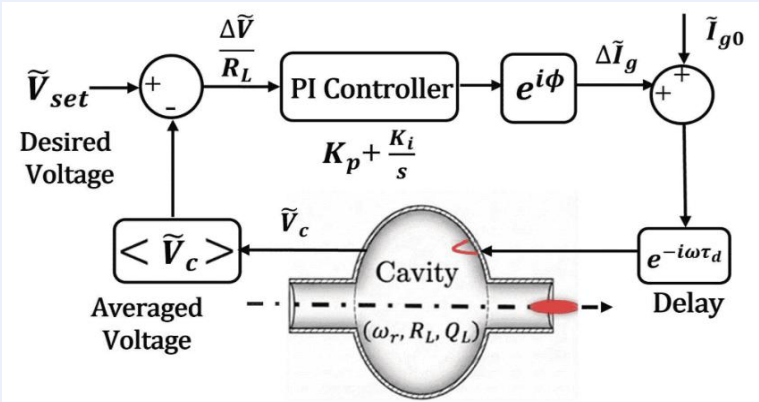
Implemented into the STABLE tracking code

<https://github.com/hetianlong-afk/STABLE>

- Dispersion equation: $\Omega^2 - \omega_g^2 = -i \frac{I_0 \alpha_c}{T_0 (E/e)} \sum_{p=-\infty}^{+\infty} [pM\omega_0 \mathbf{Z}_{tot}(pM\omega_0) - ((pM + \mu)\omega_0 + \Omega) \mathbf{Z}_{tot}((pM + \mu)\omega_0 + \Omega)]$

Total cavity impedance = closed-loop main-cavity impedance + open-loop harmonic-cavity impedance

- Open-loop cavity impedance: $Z_{mc}^{ol}(\omega) = \frac{R_{mc}}{1 + iQ_{mc}(\omega_{r,mc}/\omega - \omega/\omega_{r,mc})}$ $Z_{hc}(\omega) = \frac{R_{hc}}{1 + iQ_{hc}(\omega_{r,hc}/\omega - \omega/\omega_{r,hc})}$



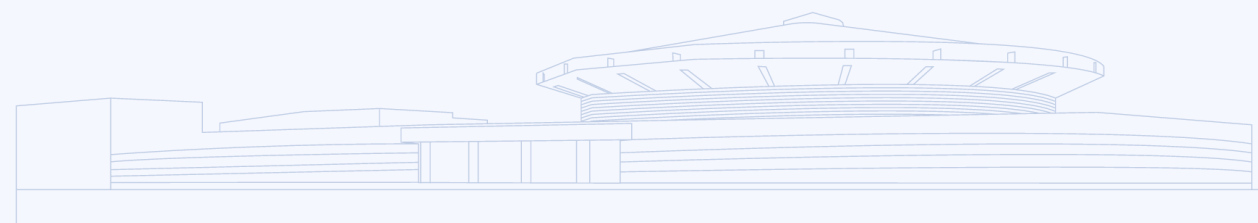
- Closed-loop main-cavity impedance:

$$Z_{mc}^{cl}(\omega) = \frac{Z_{mc}^{ol}(\omega)}{1 + T_{eff}(\delta\omega)Z_{mc}^{ol}(\omega)}$$

- Effective open-loop gain function:

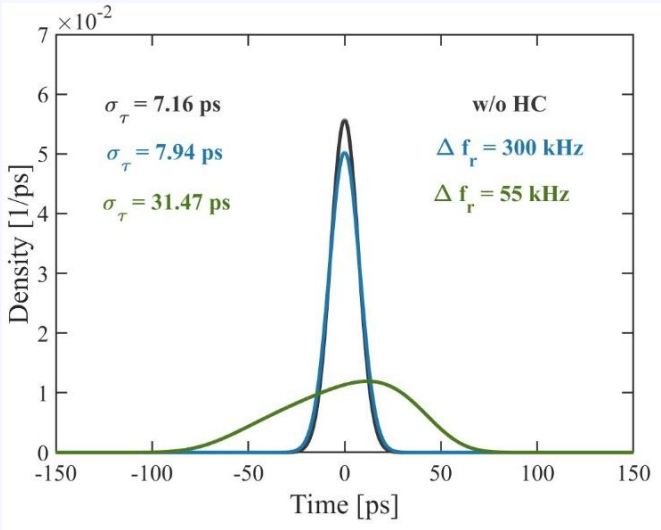
$$T_{eff}(\delta\omega) = H_{avg}^2(\delta\omega)G(\delta\omega)e^{i\delta\omega(T_d+T_b)} \quad \delta\omega = \omega - \omega_{rf}$$

- Two methods are introduced: a dispersion equation (single-macroparticle model) and a tracking simulation (multi-macroparticle model). Both include cavity beam loading and main-cavity LLRF feedback.
- Next, Let's see Mode 0 instability results for HALF from both methods.



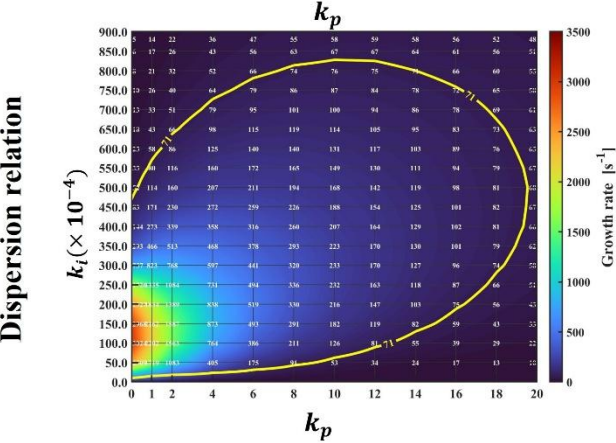
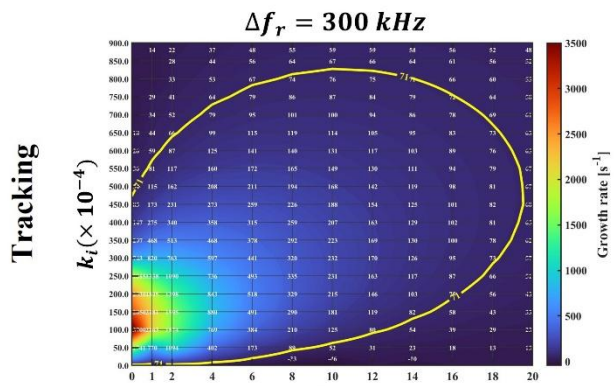
Case of HALF

- PI feedback parameter scan (k_p, k_i) → Mode 0 instability occurs over a fairly wide parameter range
- This Mode 0 instability mainly arises from improper selection of the MC PI feedback parameters and is independent of the HC operation.

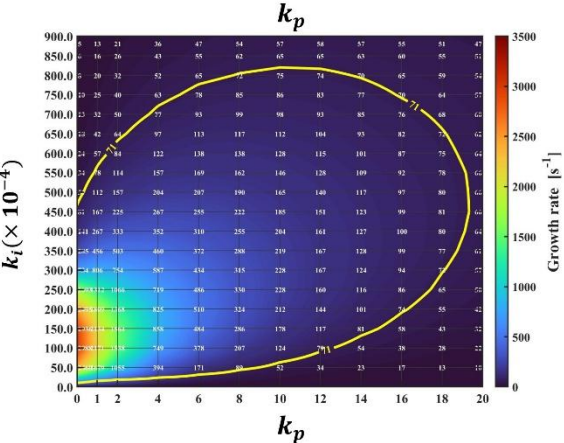
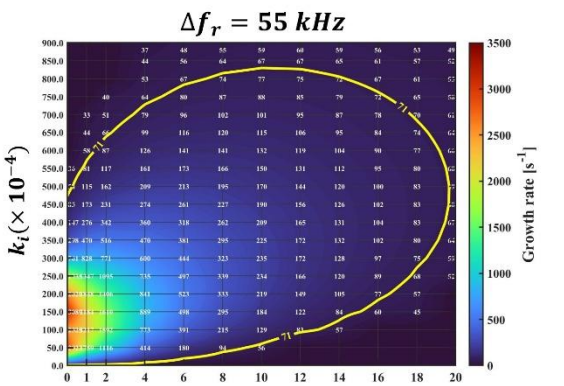


HALF @350 mA	R/Q [Ω]	QL	Δfr [kHz]
Main cavity	45	1.1e5	-7 @1.2 MV
Harmonic cavity	40.5	4e8	300/55

Almost no bunch lengthening

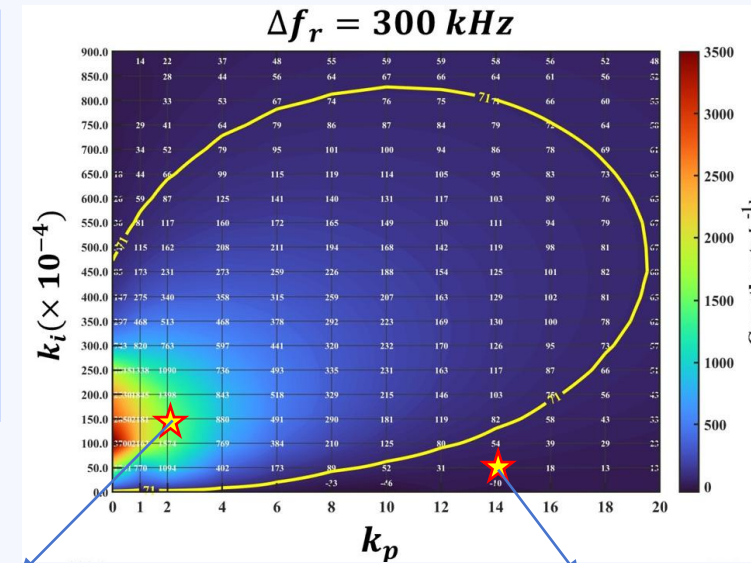


4x bunch length

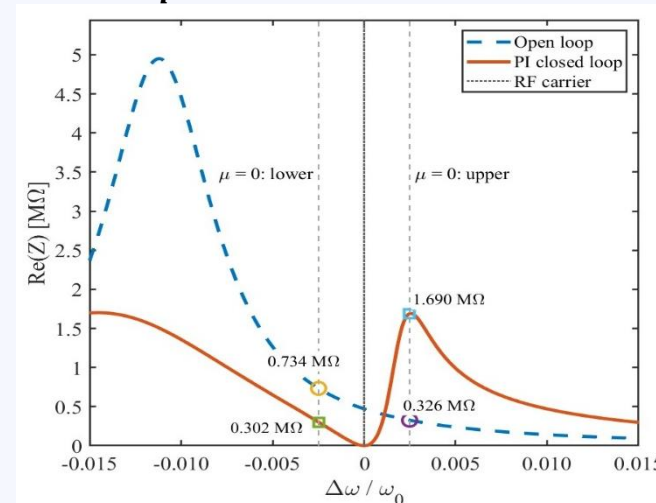


Why PI gains affect the zero-mode instability?

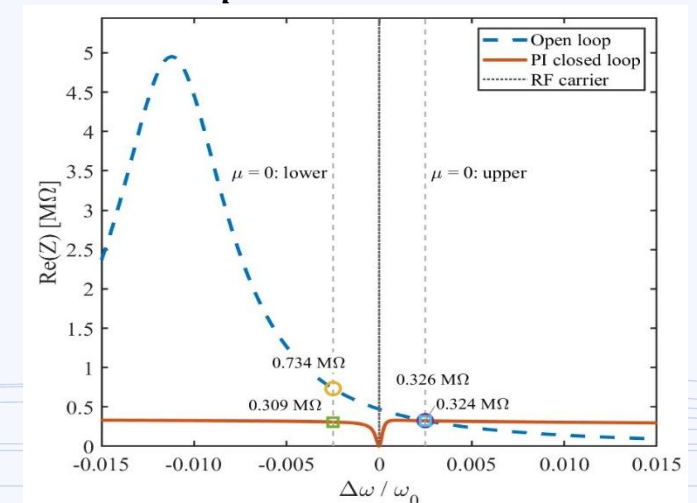
- Select two parameter settings:
- With PI feedback:
Impedance behavior near Mode 0 changes
→ from damping to excitation of Mode 0



$k_p = 2, k_i = 0.015$



$k_p = 14, k_i = 0.005$



- Two types of Mode 0 instabilities have been introduced.
- One is driven by the high- Q_L impedance of a superconducting passive harmonic cavity. Its special feature is that it is enhanced by radiation damping.
- The other is unrelated to the harmonic cavity and is caused entirely by improper PI feedback parameters of the main cavity. It can be avoided by choosing appropriate PI gain parameters, or by adopting a dedicated Mode 0 damper like the one at MAX IV.

Thanks for your attention !

